ON THE TOTAL TEMPERATURE SENSITIVITY OF CONSTANT-TEMPERATURE ANEMOMETERS

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1. Introduction

In this paper we review some results concerning the total temperature sensitivity of constant-temperature anemometers in supersonic flows. In particular, the non-linear response at low overheat ratio, first presented by Smits & Perry [1] and Smits et al. [2] is discussed.

2. CTA sensitivity to total temperature fluctuations

At supersonic velocities, a CTA is sensitive to both mass flow (ρu) and total temperature (T_0) fluctuations [3]. Therefore, a first order small perturbation analysis of the output voltage e yields:

$$e' = k_{\rho u} \cdot (\rho u)' + k_{T_0} \cdot T_0' \quad , \tag{1}$$

where $k_{\rho u}$ and k_{T_0} are the anemometer's sensitivity coefficients to mass flow and total temperature fluctuations, and the prime denotes fluctuating quantities. When the overheat parameter $\tau = (T_w - \eta T_0)/T_0$ and the total temperature are constant (η is the wire recovery factor), $k_{\rho u}$ is obtained by plotting the anemometer output voltage as a function of the mass flow following the semi-empirical relation [2]:

$$e^2 = L(T_0, \tau) + M(T_0, \tau) \cdot (\rho u)^n , \tag{2}$$

which yields:

$$k_{\rho u} = S_{\rho u} \cdot \frac{e}{\rho u} = \frac{n \cdot M}{2 \cdot e} (\rho u)^{n-1}, \tag{3}$$

where $S_{\rho u}$ is the non-dimensional mass flow sensitivity. It can be seen in (3) that $k_{\rho u}$ and $S_{\rho u}$ are function of the three variables τ, T_0, and ρu. However, L is usually small so that $S_{\rho u}$ can be approximated by $S_{\rho u} \sim n/2$.

Since an independent variation of the total temperature with constant mass flow is often difficult, k_{T_0} is usually deduced from the experimental value of $k_{\rho u}$ obtained using the precedent calibration procedure. Indeed, differentiating (2) with respect to T_0 leads to the following relation for the total temperature sensitivity [2, 4]:

$$k_{T_0} = S_{T_0} \cdot \frac{e}{T_0} = \frac{e}{2 \cdot T_0} \left(a - \frac{\eta}{\tau} - 2bS_{\rho u} - \frac{\tau + \eta}{n} \left[\left(n - 2S_{\rho u} \right) \frac{f'(\tau)}{f(\tau)} + 2S_{\rho u} \frac{g'(\tau)}{g(\tau)} \right] \right) , \tag{4}$$

where a, b, $f(\tau)$, and $g(\tau)$ are defined in [2]. It can be seen that S_{To} doesn't depend directly on T_0 and ρu. These dependences are hidden in $S_{\rho u}$ and τ. In particular, when $S_{\rho u}$ is assumed constant, S_{To} depends only on τ.

This definition of k_{T_0} takes into account the variation of the flow properties with total temperature (constants a and b) and the variation of heat transfer with overheat ratio (functions f and g). When these parameters are neglected and when the recovery factor η is assumed to be unity, equation (4) degenerates in the following simple relation, which is generally used in sub-sonic flows [5]:

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Abstract

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$$S_{T_0} = -\frac{1}{2\tau}. \quad (5)$$

Figure 1 shows a plot of S_{To} for the two cases (subsonic and supersonic), using the calibration data of Kovásznay [3]: η=0.95, a=b=0.768, $f(\tau)$=$g(\tau)$=1-0.18τ, n=0.5, $S_{\rho u}$=$n/2$=0.25. The diagram in the inset show the same curves with a zoom on the y-coordinate: it shows that equation (4) leads to negative values of S_{To} when τ is high enough (τ > 1). This is physically impossible and the whole relation breaks down at high overheat, because of wire burnout. The problem arising at low overheat ratio is clearly seen in figure 1: a small variation of T_0 produces a variation of τ if T_w is maintained constant ; therefore, since S_{To} is a strong function of τ at low overheat, the instantaneous sensitivity can differ significantly from its value at the average total temperature, thus compromising the linear based modal analysis [2].

3. Non-linearity of the response to total temperature fluctuations

A more quantitative approach to this issue can be obtained by developing the anemometer's response to total temperature fluctuations. If the mass flow over the wire is supposed constant, a variation ΔT_0 of the total temperature results in a variation Δe of the output voltage:

$$\Delta e = k_{T_0} \Delta T_0 + R(\Delta T_0) \ , \quad (6)$$

where R consists of terms of order greater than one. If the system were perfectly linear, k_{T_0} would be constant and R would be zero. This is in reality not the case, but when ΔT_0 is sufficiently small, R is negligible and a local linearization approach (ie a first order analysis) may be sufficient. However, since k_{T_0} is (like S_{To}) a strong function of τ (and consequently of T_0) at low overheat, higher order terms may have an influence even for reasonably small variations of T_0, thus compromising the first order analysis. Equation (6) can be written in the form $\Delta e = k_{T_0} \Delta T_0 (1+P)$, where $P = R(\Delta T_0) / k_{T_0} \Delta T_0$ is a measure of the error induced by neglecting a finite value of R. When P is positive, the total temperature variation is overestimated.

If a known heat transfer relation is assumed, P can be obtained for a given value of ΔT_0 by computing the actual variation Δe of the anemometer output voltage using equation (2), as well as the temperature sensitivity k_{T_0} using (4) (algebraic relations for L and M are given in reference [6]). Results showing the variation of P as a function of the overheat ratio are presented in figure 2. As before, the calibration data of Kovásznay has been used. The curves were obtained by computing the difference $\Delta e = e(\overline{T_0} + \Delta T_0 / 2) - e(\overline{T_0} - \Delta T_0 / 2)$ and comparing it with $k_{T_0}(\overline{T_0}) \cdot \Delta T_0$ to obtain R and then P. The mean total temperature $\overline{T_0}$ is 293K and two values of ΔT_0 have been chosen: $\Delta T_0 = 48K$ corresponds to a variation of +/– 2σ where σ=4% is a typical total temperature RMS fluctuation in a supersonic flat plate turbulent boundary layer [2]. In this case, P stays lower than 1% only in a limited domain of overheat ratios comprised between τ=0.3 and τ=0.9. A full modal analysis down to $\tau<0.1$ appears therefore to be impossible, as suggested by Smits & Dussauge [7]. On the other hand, for ΔT_0=12 K (which corresponds to a variation of +/– 2σ where σ=1%), the error appears to be acceptable down to τ=0.05.

According to this analysis, investigation of turbulent flows involving very low total temperature fluctuations is therefore possible with a constant temperature anemometer. This is for example the case in the free-stream of supersonic wind tunnels, where very small reminiscent "entropy type" disturbances may be superimposed to acoustic waves (the same conclusion has been reached in reference [8], albeit with a slightly different method). These results show however that when the total temperature fluctuation reaches several percent, a small perturbation analysis of the output voltage fluctuation is insufficient, as argued by Smits & Dussauge [7].

4. Influence of the feedback system

The analysis presented in sections 2 and 3 were made assuming that the feedback system imposes a constant value of the wire temperature *Tw*. In reality, the wire resistance has to vary by a small amount to give the amplifier a finite feedback signal, so that the assumption of constant wire temperature is only an approximation.

The static analysis of a non-ideal CTA has been performed by Perry & Morrison [9]: in essence, the operating resistance *Rw* of the wire is given by the intersection of two equations:

- The equation relating the output voltage to the flow parameters ; this is the square root of equation (2):

$$e = \sqrt{L(R_w) + M(R_w)\cdot(\rho u)^n} \tag{7}$$

- The equation obtained by an analysis of the electrical circuit [9]:

$$e = \frac{(R_a + R_w)\cdot(R_b + R_c)\cdot E_{qi}}{(R_a + R_w)\cdot(R_b + R_c) + K_{dc}\cdot(R_w\cdot R_c - R_a\cdot R_b)} \tag{8}$$

where R_a, R_b, and R_c are the resistances of the Wheatstone bridge, K_{dc} is the amplifier's DC gain, and E_{qi} is the anemometer's offset voltage[1].

The wire operating point is best represented graphically by the intersection of the curves (7) and (8) on a (e, R_w) plane: this is presented in figure 3 for an overheat parameter of approximately $\tau \sim 0.3$ (the feedback amplification is $K_{dc}=5000$). It can be seen that the wire resistance is always higher than the resistance corresponding to a perfectly balanced bridge, and that it depends on the offset voltage E_{qi}: the higher the offset voltage, the higher the wire resistance. In particular, it is important to notice than a very high DC gain is not sufficient to balance the bridge. As shown by Perry & Morisson [9], the Wheatstone bridge would be perfectly balanced only for $K_{dc} \to \infty$ and $E_{qi}=0$. For the present case, taking values of K_{dc} larger than 5000 has a negligible influence on the operating point. The offset voltage has a large influence on the anemometer's dynamic performance and typical values of E_{qi} range between 0.1 mV and 30 mV, depending on the anemometer's characteristics.

Since the curve corresponding to equation 8 is not vertical, the wire resistance varies slightly with the temperature and the actual sensitivity of the real anemometer is therefore lower than the one obtained by considering a sensor maintained at a constant resistance. The deviation from an ideal anemometer is a function of the offset voltage and of the overheat ratio. At very low overheat, the temperature sensitivity decreases drastically, as shown in figure 3 as well as in the figure 3 of reference [1].

Consequently, the total temperature fluctuation measured with a real system by using the sensitivity obtained with equation 4 or 5 will be under-estimated, particularly at low overheat and high offset voltage. This is illustrated in figure 4, where the error *P* is computed by taking this effect into account. For the seek of simplicity, fluid property variations are ignored (i.e., $a=b=0$ and $f(\tau)=g(\tau)=0$).

In opposition to the results obtained in section 3, *P* appears to be independent on the fluctuation level as long as E_{qi} is not too small. For $E_{qi}=0$, the system behaves like a perfect constant temperature bridge and *P* has the same value as in figure 2. For $E_{qi}>0$, the total temperature fluctuations appear to be underestimated, and the relative error is significant even at high overheat ratio.

It should be noted that the same phenomenon happens for the sensitivity to mass flow fluctuations. The real anemometer sensitivity is lower than the ideal one because of the finite offset voltage. In this case, however, the deviation is taken into account during calibration since the mass flow sensitivity is obtained by recording the output voltage of the real anemometer as a function of the mean mass flow, so that the operating point follows curve (8) during the

[1] A similar expression for a CTA with offset current has been derived in reference [8].

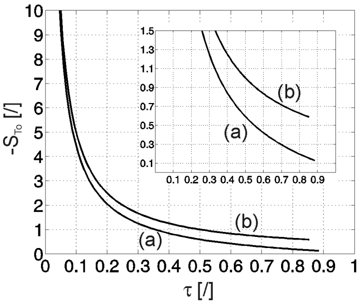

Fig. 1: CTA Total temperature sensitivity
(a) supersonic case (eq. 4)
(b) subsonic case (eq. 5)

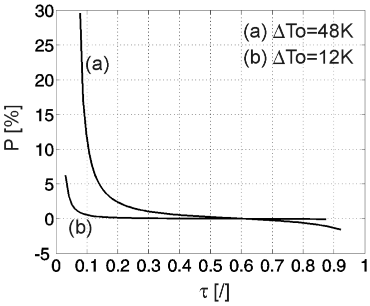

Fig. 2: Response non-linearity (ideal CTA)

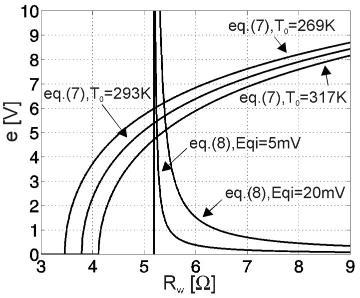

Fig. 3: CTA operating point

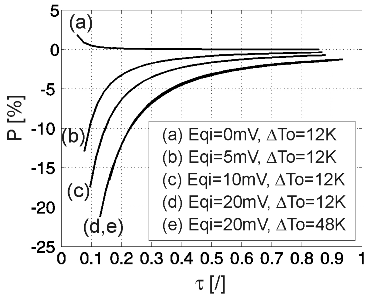

Fig. 4: Error due to the finite offset voltage

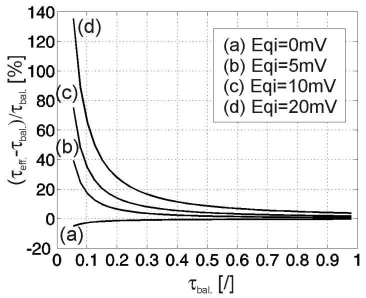

Fig. 5: Error in the overheat estimation

calibration procedure. As pointed out by Perry & Morrison [9], the effect of wire resistance variation with the mean mass flow is therefore to change the coefficient M in equation (3) so that the gradient of the calibration curve deviates from the ideal constant temperature value. The measured slope $M(T_0, \tau)$ is smaller than the one which would be obtained with an ideal anemometer and account for the decrease in $k_{\rho u}$, as can be seen by considering equation (3). This has however no effect on the total temperature sensitivity, as shows equation (5).

These results show that a derivation of k_{T_0} assuming a constant wire temperature can lead to serious errors when total temperature fluctuations are measured, regardless of the level of these fluctuations. Strictly speaking, these errors are not due to the non-linear heat transfer relation, but to the inability of the feedback loop to maintain a constant wire temperature. To avoid these errors, a direct calibration of the CTA at different total temperatures should be performed. This requires a facility allowing independent variation of the mass flux and the total temperature. Another alternative would be to use an anemometer with frequency dependent gain, where the offset voltage has no direct influence on the dynamic behaviour and can be maintained very small [10].

5. Influence of the offset voltage on the overheat ratio

It has been showed in the last section that the wire resistance in a real CTA is always slightly higher than the resistance corresponding to a perfectly balanced bridge: $(R_w)_{\mathrm{balance}}=(R_a \cdot R_b)/R_c$. This means that the exact wire overheat depends on the flow conditions and E_{qi} and is not known *a priori*. Very often, the exact determination of the wire overheat is not of principal importance if the measurements are performed at the same settings as during calibration, but when the fluctuation diagram technique is used, the wire overheat becomes an important parameter and its determination should be made accurately. This is especially the case when the anemometer allows an offset variation for frequency optimisation purpose: since measurements at supersonic velocities require a careful adjustment of the dynamic response, a change of E_{qi} can usually not be avoided during the tests.

The relative error which is made by assuming a perfectly balanced bridge is presented in figure 5. The real (or "effective") overheat parameter is denoted τ_{eff} whereas the overheat corresponding to a perfectly balanced bridge is written τ_{bal}. Since k_{T_0} is a strong function of the overheat, neglecting this error can lead to an overestimation of the sensitivity. To overcome this problem, the authors propose to record the anemometer output voltage <u>and</u> the wire voltage simultaneously, in order to be able to compute the effective wire overheat.

6. Conclusion

It has been showed that the non-linear relationship between total temperature variations and the output voltage of a CTA can induces large errors at low overheat ratios, as already discussed by Smits & Perry [1] and Smits et al. [2]. This is however restricted to relative fluctuations of several percents, typical of fully developed turbulent flows. For a lower fluctuation level, like in the free stream of supersonic wind tunnels, a first order approximation seems accurate enough to perform a full modal analysis. Of more concern is the overestimation of total temperature sensitivity that occur when performing only a direct mass flow calibration and deriving a relation assuming constant wire-resistance, since it appears to be independent of the fluctuation level.

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